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Evolution theory for optimal control of a Couette iceform design model

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Abstract—The 'iceformation' method is a new technique for designing the shape of low-loss flow fixtures and bodies. Optimal selection of the flow and thermal control parameters is based on minimizing energy dissipation when the ice/water interface reaches steady state. A theoretical basis for selecting control parameters is established. An optimal criterion is derived using a Couette iceform model for a point on the ice/water interface and an evolution theory. The evolution process consists of dynamic variation and thermodynamic performance evaluation. The paper provides a theoretical foundation for conducting iceformation experiments and numerical simulation of analogous design processes.

1. INTRODUCTION

A LARGE component of fluids and thermal design is the determination of boundary geometry. The shape of a device's boundary largely determines its fluid dynamics, drag, heat transfer and thermodynamic performance. Any mechanism, natural or human produced, which changes the surface geometry also designs the shape. The 'iceformation' method uses the natural process of ice formation as a tool to design a flow's boundary shape [1]. The flow directly alters the surface geometry. At the same time the changing surface geometry alters the flow. This mechanism can be used to design bodies or fixtures that contain complex flow and heat transfer, but in order to produce a useful shape, the natural process must be controlled and guided along the desired path of performance improvement.

1.1. *Iceformation design feasibility*

Previous work shows the feasibility of the iceformation design idea and provides a basis for studying ice formation as a design tool. Carlson [2] produced and tested cylindrical iceforms for drag reduction, but the results were inconclusive. Similarly, in traditional ice formation studies, Maksimov [3] and Cheng *et al.* [4] generated iceforms for a cylinder in crossflow. Although the drag performance was not evaluated, the shapes had features that indicated a reduced wake.

An experimental study by LaFleur [1] highlights the feasibility of using iceformation as a design tool for diffusers. In testing the hypothesis of loss reduction, experimental evidence showed that iceform diffusers had lower loss than did equivalent cones of the same length and area ratio. Also, a new concept diffuser, called a 'ring' diffuser, was generated. In another study LaFleur and Langston [5] used the iceformation method to design a three-dimensional contour geometry for a cylinder/flat plate juncture. A test contour reduced the horseshoe vortex and wake drag by an average of 18%.

In summary, these studies show the need for a theoretical basis for reduction of energy dissipation and a connection between resulting geometries and specified control parameters. The purpose of this work is to establish an optimal design criterion for utilizing the ice formation process to design bodies and fixtures that are appropriate for complicated flow and thermal situations. A theoretical basis for the iceformation design tool is derived using an evolution theory and a fundamental Couette iceform model.

1.2. *Motivation for the Couette iceform model*

A new theoretical approach should use a fundamental problem as a first step. The Couette iceform model provides this first step in developing a theoretical basis for iceformation design. The innovations offered in this paper are not overwhelmed by the complexities of an unsteady three-dimensional design problem such as a cylinder/hull juncture [5]. The Couette iceform model can be quickly understood. The Couette model offers a fundamental regime to test the loss reduction hypothesis of iceformation design. More complicated flow regimes will be studied in subsequent investigations.

To use the iceformation method to design a shape, first the cold parent surface must be designed. A preprocess procedure yields guidelines for selecting flow and thermal control parameters. A preprocess model would be useful for designing an iceform apparatus by approximating the flow, thermal and geometric characteristics for selected ice/water interface points such as the point shown in Fig. 1. The point ice geometry or experimental conditions are controlled. In between the control points, the geometry is sculpted

NOMENCLATURE

Br	Brinkman number, $Pr\ Ec$
C	specific heat
d	designation function
Ec	Eckert number, $\equiv U_F^2/C(T_F-T_0)$
f	performance functional
g	generation function
h	film coefficient
I	interface position
J_q	heat flux
k	thermal conductivity
L	evolution length
m	J_q flexible number
n	f flexible number
Nu_I	Nusselt number based on the water space, $\equiv h(\delta - I)/k_W$
Pr	Prandtl number, $\mu C/k_W$
Q	latent heat of formation
t	time
T	dimensional temperature
T_y	normal temperature gradient, $\partial T/\partial y$
u	tangential velocity profile
u_y	normal velocity gradient
v	nondimensional velocity
y	parent shape normal coordinate.

Greek symbols

β	dissipation strength
δ	water channel width
Φ_T	total performance indicator
Φ	internal performance indicator
κ	conductivity ratio, k_W/k_I
ξ	nondimensional water space, $(1-\zeta)$
λ	nondimensional water space coordinate
μ	absolute viscosity
ν	kinematic viscosity
ψ	nondimensional ice space coordinate
Ω	real constraint limit function
ρ	density
P	formation rate function
σ	entropy production
τ_I	nondimensional ice temperature, $(T_I - T_R)/(T_0 - T_R)$
τ_W	nondimensional water temperature, $(T_W - T_0)/(T_F - T_0)$
θ	heat transfer strength
θ_T	temperature ratio, $T_F - T_0/T_0 - T_R$
ζ	nondimensional interface position, I/δ.

Subscripts and superscripts

F	moving plate boundary
O	interface boundary
R	cold parent surface boundary
I	ice field
W	water field
S	steady state
M	minimum dissipation
$\otimes$	selected pair of parameters
0	initial value.

ice formation $\equiv$ mechanism
iceformation $\equiv$ design method.

according to the flow patterns and ice interface interactions similar to a flexible string held at its end points [6]. For example, Fig. 1 indicates the Couette model applied to a control point that defines frontal area.

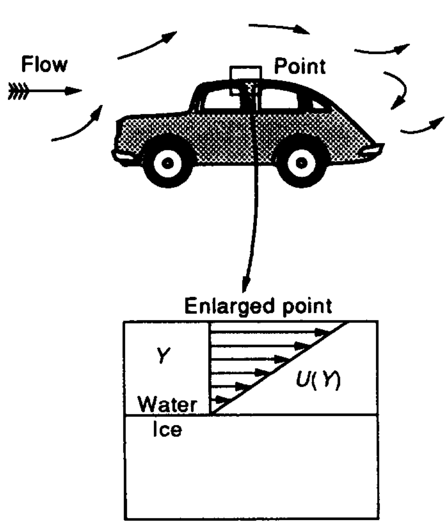

FIG. 1. Ice/water interface point.

The Couette model of iceformation provides the surface shear stress and temperature gradients typical of a point on an ice/water interface. Any point on an interface in a flow can be modeled as a Couette flow provided characteristic velocity and plate spacing can be identified. This holds true for a flow with relatively large Prandtl number such as water and flows with pressure gradients.

For example, the turbulent boundary layer contains a laminar sublayer. Near the boundary, a linear profile approximates the velocity profile. The universal turbulent boundary layer velocity profile indicates characteristic velocity and laminar sublayer thickness. Bejan [7] showed that the dissipation is concentrated in the near wall region. Laminar boundary layers are represented by series solutions. The near wall region can be approximated by a linear velocity profile due to dominant terms in the series as the wall is approached. Even a Couette flow with a pressure gradient can be represented by a smaller Couette flow with a varying characteristic plate velocity. The Couette model approximates the limiting flow field and ice field in the near interface region.

A schematic diagram of the one-dimensional

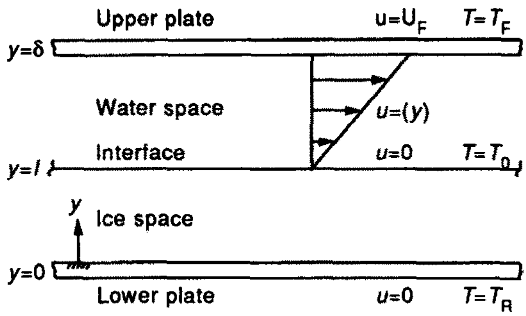

FIG. 2. Iceformation in a Couette flow regime.

Couette iceform model is shown in Fig. 2. The lower stationary plate, called the parent shape, is cooled below the freezing point of water ($T_R < T_0$) while the moving upper plate is maintained at a temperature above the freezing point ($T_F < T_0$). The external constraints to the shape design process are the upper plate velocity and temperature, as well as the lower plate temperature and plate separation distance (δ). The internal material constraints are water thermal conductivity, ice conductivity, water viscosity, ice density, phase change temperature and latent heat of formation. The design mechanism of phase change causes movement of the ice surface geometry, which is tracked by $y = l(t)$. The flow and thermal fields can be nondimensionalized in terms of the water and ice spaces and the boundary conditions. Appendix A summarizes the model assumptions and the well-known flow and thermal solutions.

1.3. *Design evolution strategy*

An evolution theory provides a basis for controlling the iceformation design process using the Couette iceform model. Evolution is the process of variation and selection and therefore, it must be *both* dynamic and thermodynamic [8, 9]. Methods of design are distinguished by the techniques for geometric variation and performance based selection.

Traditional design methods have human specified variation and selection, and there is a high reliance on the designer's understanding of the flow and intellect to produce good variations. In an attempt to quicken the design process, some design optimization methods have biological or genetic aspects. Eigen [10] advocated using a 'genetic algorithm' approach to experimental design of optimal fluid passages. In his study, many cycles of random geometric variation and subsequent performance evaluation (selection) were needed for optimization. It is interesting to note that Eigen's optimal expansion nozzle result has a number of ring-like structures, much like Gilpin's iceform pipes [11] and LaFleur's iceform ring diffuser [1]. French [12] discussed the institution of natural design features into engineering design problems rather than simulating geometric variation and selection, and showed that this approach is useful for a variety of design problems and constraints.

In contrast to Eigen's and French's as well as to the traditional approaches, the iceformation method utilizes natural forces to perform geometric variations while simultaneously evaluating performance. The process is controlled and guided along the path of performance improvement by human selection of constraints. Figure 3 shows that the design process responsibilities are split such that the geometric variations are produced by the natural physics and the selection of good variants is controlled by the designer.

The loss reduction hypothesis [2, 1, 6] states that the natural process of iceformation produces shapes of lower energy dissipation. When opposite forces driving a process equilibrate, the configuration reaches a 'happy medium'. Steady state and optimization are similar in that respect [9]. In this paper, an evolution theory identifies the difference between steady state geometry and the optimum geometry of minimum energy dissipation. Then a criterion for the selection of experimental flow and thermal parameters is formulated such that steady state iceform geometries are selected to minimize energy dissipation. The ice formation process can be controlled and used as a flow and thermal design tool.

1.4. *Theoretical foundation and goals*

This paper presents two independent theories of the ice formation process: dynamic, which describes design variation, and thermodynamic, which describes design selection. An evolution theory combines the variation and selection formulations. The theoretical foundation of the iceformation method consists of function definitions, control variables and equations which lead to satisfaction of the loss reduction hypothesis.

The variation theory describes the interface dynamics utilized for shape variation. The shaping of the ice interface is a dynamic process of geometric variations caused by a phase change mechanism. For the Couette model, the geometry of the ice/water interface is related to the adjustable specified control parameters, the Brinkman number Br, a thermal parameter θ_T and time or $\zeta(Br, \theta_T, t)$. The Brinkman number is

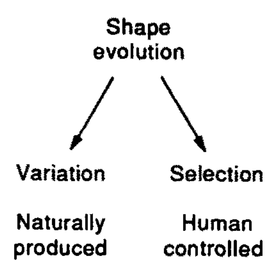

FIG. 3. Splitting evolution.

not zero for the Couette iceform model. When steady state is reached, $\zeta \to \zeta_S$, and the steady state geometry is related to the specified constraints, $\zeta_S(Br, \theta_T)$. The goal of the variation theory is to obtain explicit relationships for $\zeta_S(Br, \theta_T)$.

The selection theory provides a thermodynamic formulation to judge iceform shape performance. The ice formation process contains thermal and flow processes which dissipate energy. Energy dissipation is related to the geometry and the specified control parameters, $\Phi(\zeta, Br, \theta_T)$. The minimization of energy dissipation implicitly provides a set of optimal geometries that depend on the flow and thermal parameters. The goal of the selection theory is to obtain an explicit relationship between performance and optimum geometries in terms of constraints, $\zeta_M(Br, \theta_T)$ and $\Phi(\zeta, Br, \theta_T)$.

The goal of the evolution theory is to establish a practical criterion for selecting experimental flow and thermal constraints, Br and θ_T, in order to produce steady state geometries that minimize energy dissipation. This provides a theoretical foundation for using ice formation to design low energy dissipation bodies and fixtures.

2. VARIATION THEORY: ICE FORMATION DYNAMICS

The natural process of ice formation actively designs fluid dynamic shapes by variation of interface geometry with time. Much work has been completed describing the nonlinear dynamics of the ice formation mechanism (both melting and growing). Yao and Prusa [13] provide an extensive review of traditional work on the ice formation mechanism, including forced convection studies. These studies are useful in describing iceform shape variation.

The rate of formation is described by a well-known interface equation which connects the phase change with interface heat fluxes using an energy balance about the infinitely thin interface. The nondimensional geometry version of the one-dimensional interface equation is

$$\frac{\partial\zeta}{\partial t} = \left[\frac{k_I(T_0 - T_R)}{Q\rho_I\delta^2}\left(\frac{d\tau_I}{d\psi}\right)_{\psi=1}\right]\frac{1}{\zeta} - \left[\frac{k_W(T_F - T_0)}{Q\rho_I\delta^2}\left(\frac{d\tau_W}{d\lambda}\right)_{\lambda=0}\right]\frac{1}{1-\zeta} \quad (1)$$

where $\zeta \equiv l/\delta$. Stephan [14] used ξ, the nondimensional liquid space, to track the interface and plotted results in terms of $(1-\xi)$, the nondimensional solid space. Here the nondimensional ice space, ζ, is used instead of ξ, because the formation is supported by the cooled parent surface and this is where the origin of ice growth is located.

2.1. *Heat transfer strengths*

The heat transfer terms from the ice and water phases have an opposite effect on the interface movement and are classified as 'opposite design forces' [15] for the geometry, ζ. The 'strength' of each phase to occupy space is indicated by the bracketed terms of equation (1), and these are defined as positive definite quantities called 'heat transfer strengths'. Utilizing the well-known thermal and flow solutions summarized in Appendix A, the Couette ice heat transfer strength for geometric growth is defined by equation (2)

$$\theta_I \equiv \frac{k_I(T_0 - T_R)}{Q\rho_I\delta^2}\left(\frac{d\tau_I}{d\psi}\right)_{\psi=1} = \frac{k_I(T_0 - T_R)}{Q\rho_I\delta^2}. \quad (2)$$

The water heat transfer strength for geometric decay is defined, evaluated and stated in terms of the Brinkman number in equation (3)

$$\theta_W \equiv \frac{k_W(T_F - T_0)}{Q\rho_I\delta^2}\left(\frac{d\tau_W}{d\lambda}\right)_{\lambda=0} = \frac{k_W(T_F - T_0)}{Q\rho_I\delta^2}Nu_I \quad (3)$$

where the Couette water yields

$$Nu_I = \left[1 + \frac{Br}{2}\right]. \quad (4)$$

To use the Couette iceform model for more complicated flow and ice fields, the Nusselt number based on the water space can be related to heat transfer correlations of Nusselt number based on streamwise coordinates such as

$$Nu_I = \left(\frac{d\tau_W}{d\lambda}\right)_{\lambda=0} = Nu_I(Nu_x) = Nu_I(Re_x, Pr). \quad (5)$$

The heat transfer strengths may become a function of streamwise and cross-stream coordinates and ice interface position.

The heat transfer strengths are opposing design forces which result in a steady state. If one strength is missing (θ_I or θ_W), the resulting steady state is all liquid or solid as shown by Stephan [14], Gupta and Kumar [16], and Yao and Cherney [17].

2.2. *Generation function phase plane: steady state*

With the rate of formation as a rate of geometric change

$$\mathrm{P} \equiv \frac{\partial\zeta}{\partial t} \quad (6)$$

the interface equation can be written as a generic formation equation

$$\frac{\mathrm{P}}{\theta_I} = \frac{1}{\zeta} - \frac{\theta_W}{\theta_I}\frac{1}{1-\zeta} \quad (7)$$

which can be solved analytically if the heat transfer strengths are constant. The heat transfer strength ratio is a governing parameter for the ice formation process. The formation rate can be plotted as

$$\frac{\mathrm{P}}{\theta_I}\left(\zeta, \frac{\theta_W}{\theta_I}\right),$$

as shown in the mid to lower part of Fig. 4. Figure 4 shows that steady state is reached when the heat transfer strength ratio curves cross the interface position axis (zero rate). These steady states are stable in time due to the negative slope of the curves. Figure 4 also shows that the formation rate is theoretically infinite at $\zeta = 0$, full water space, and $\zeta = 1$, full ice space. Another function, the generation function, describes the ice formation dynamics without the infinite values. The generation function is defined as

$$g(\zeta) \equiv \zeta(1-\zeta)\mathrm{P} = \zeta(1-\zeta)\frac{\partial \zeta}{\partial t}. \qquad (8)$$

The formation equation (7), can be written in terms of the generation function (8), divided by the parent surface generation magnitude, $|g(0)|$

$$\frac{g(\zeta)}{\theta_\mathrm{I}} = 1-\zeta\left[1+\frac{\theta_\mathrm{W}}{\theta_\mathrm{I}}\right]. \qquad (9)$$

The generation function is a transformation of the formation rate, and for the Couette iceform model it is a linear transformation of geometry, ζ. The generation function forms a phase plane with the interface position. Figure 5 shows the generation phase plane process lines and steady state vs the heat transfer strength ratio. At steady state, $\zeta = \zeta_\mathrm{S}$, the generation function is zero, $g(\zeta_\mathrm{S}) = 0$. The unique steady states appear on the phase diagram (Fig. 5), when lines of constant heat transfer strength ratio $\theta_\mathrm{W}/\theta_\mathrm{I}$ cross the $g = 0$ axis. The Couette iceform steady states are dynamically stable (with time progression) due to the negative slope of the heat transfer strength ratio lines on the phase plane. The steady state geometry can be stated in terms of the heat transfer strength ratio by solving for the root of $g(\zeta_\mathrm{S}) = 0$ as

$$\zeta_\mathrm{S} = \frac{1}{1+\dfrac{\theta_\mathrm{W}}{\theta_\mathrm{I}}}. \qquad (10)$$

The heat transfer strength ratio is a governing parameter and can be stated for any flow and Couette flow respectively as

$$\frac{\theta_\mathrm{W}}{\theta_\mathrm{I}} = \kappa\theta_\mathrm{T}\, Nu_\mathrm{I} = \kappa\theta_\mathrm{T}\left[1+\frac{Br}{2}\right]. \qquad (11)$$

The flow and thermal control parameters are defined as follows:

$$\kappa \equiv \frac{k_\mathrm{W}}{k_\mathrm{I}};\quad \theta_\mathrm{T} \equiv \frac{T_\mathrm{F}-T_0}{T_0-T_\mathrm{R}};$$

$$Br \equiv \frac{\mu U_\mathrm{F}^2}{k_\mathrm{W}(T_\mathrm{F}-T_0)} = Pr\, Ec. \qquad (12)$$

The Nusselt number, based on the fluid space, can be used to apply the model to include convection effects.

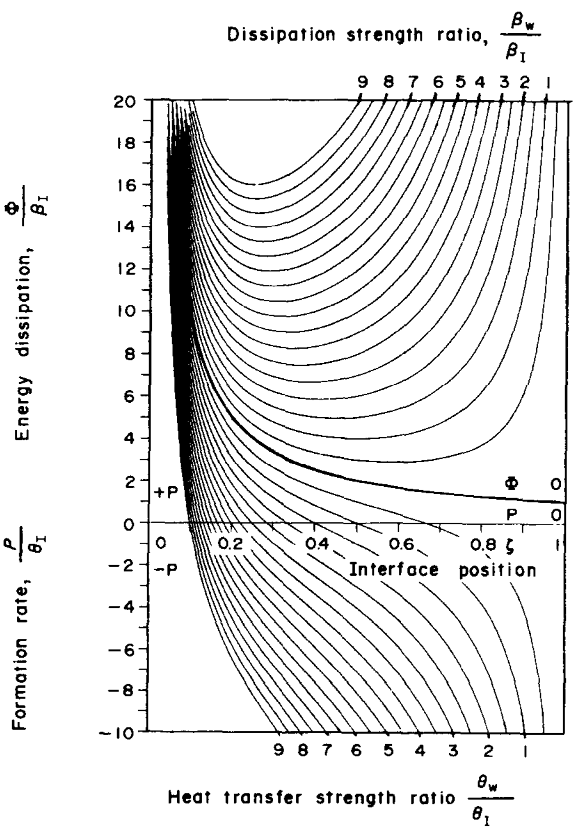

FIG. 4. Formation rate and energy dissipation vs interface position.

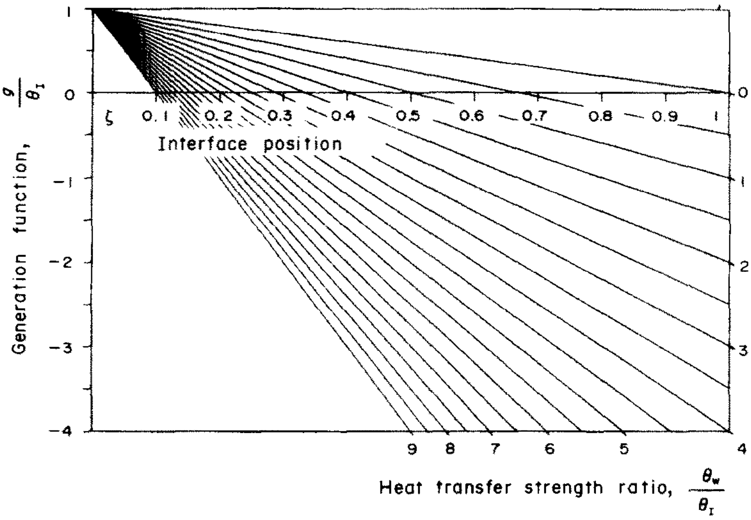

FIG. 5. Generation function phase plane.

By equations (10)–(12), the steady state geometry is connected to the specification of external constraints, θ_T, Br or T_F, T_R, U_F and the material constraint, κ. The θ_T definition is the inverse of traditional ice formation studies such as Seki *et al.* [18], Hirata and Matsuzawa [19], and Shibani and Ozisik [20]. The correlations in these papers have the convenient form of $Re\ \theta_T^n$, where $n > 0$, if θ_T is used instead of θ_C or θ. Use of the θ_T definition is justified because the design process and fluid/interface interaction are the phenomena of concern rather than just solidification. Also, the primary thermal control parameter in experiments, the water temperature, T_F, is in the numerator. Thus, as will be shown later, the use of θ_T is more convenient.

In summary, the variation theory of the Couette iceform model establishes explicit expressions for a relationship between ice geometry and the flow and thermal parameters as $\zeta_S(Br, \theta_T)$. The steady state geometry is controlled by the flow and thermal parameters. Equations (9)–(11) will be used later in the evolution theory.

3. SELECTION THEORY: THERMODYNAMIC PERFORMANCE

A measure of 'goodness' is needed to use ice formation as a design tool. The performance of the iceform shape is determined by the thermodynamic behavior of both the flow and thermal fields in the water and ice spaces. Formulation of energy dissipation as a performance indicator, Φ, or performance functional, f, allows the determination of an optimum geometry or design improvement. The optimum geometry minimizes flow and thermal energy dissipation and it depends on specified flow and thermal constraints. But the optimum geometry is not necessarily the same as the steady state geometry, and this leads to the evolution theory for constraint specification.

The energy dissipation performance functional is derived in terms of irreversible thermodynamics in the flow and thermal fields. The well-known quantity of entropy production can be formulated, as a special case, in terms of viscous dissipation and heat transfer as de Groot and Mazur [21], and Bejan [7, 22] have shown. Using entropy production as a guide to design is called 'thermodynamic design' (Bejan [23]). In the present study, the entropy production functional is made flexible and is arbitrarily stated as an energy dissipation functional. Using the calculus of variations the functional is consistent with the first law of thermodynamics by independent variation of temperature under the equation of motion constraint. The energy dissipation performance functional is derived in Appendix B as

$$\Phi(\zeta) = \int_0^\delta \left[\frac{1}{2}k\left(\frac{dT}{dy}\right)^2 + u\mu\frac{du}{dy}\frac{dT}{dy}\right]dy. \quad (13)$$

The formulation of competing viscous dissipation and thermal dissipation irreversibilities produces a convex performance functional [24] which can be optimized. For instance, Jany and Bejan [25] have formulated performance functionals for fluids and heat transfer problems, and solved for optimum geometries for a variety of configurations. Little has been done to evaluate energy dissipation characteristics of ice formation. Shape sensitivity was connected to phase change by Meric [26] based on previous work by Siegel [27]. However, Meric used the objective of heat flux or deviation from an isotherm instead of energy dissipation. Meric's geometric variations were discrete and arbitrary. The present work utilizes the natural and continuous variations of iceformation for geometric variations.

3.1. *Energy dissipation strengths*

Temperature variation occurs due to variation in the ice/water interface geometry and, subsequently, the steady state geometry is varied by variation of the external constraints θ_T and Br. The energy dissipation performance integral can be split at the interface isotherm and nondimensionalized to reveal the geometric dependence, $\Phi(\zeta)$, in a performance equation

$$\Phi = \frac{\beta_I}{\zeta} + \frac{\beta_W}{1-\zeta} \quad (14)$$

where the β terms are defined by equations (15) and (16) to be the 'energy dissipation strengths'

$$\beta_I \equiv \frac{k_I(T_0-T_R)^2}{\delta}\int_0^1 \left(\frac{d\tau_I}{d\psi}\right)^2 d\psi \quad (15)$$

$$\beta_W \equiv \frac{k_W(T_F-T_0)^2}{\delta}\int_0^1 \left(\frac{d\tau_W}{d\lambda}\right)^2 d\lambda + \frac{\mu U_F^2(T_F-T_0)}{\delta}\int_0^1 2v\frac{dv}{d\lambda}\frac{d\tau_W}{d\lambda}\,d\lambda. \quad (16)$$

The performance equation can be written in terms of the energy dissipation strength ratio

$$\frac{\Phi}{\beta_I} = \frac{1}{\zeta} + \frac{\beta_W}{\beta_I}\frac{1}{1-\zeta}. \quad (17)$$

The energy dissipation strength ratio is a governing parameter and is evaluated by the integrals of equations (15) and (16). The performance equation is similar in form to the formation rate equation (7), and is plotted as

$$\frac{\Phi}{\beta_I}\left(\zeta, \frac{\beta_W}{\beta_I}\right),$$

where the dissipation strength ratio parameter plays a parallel role to the heat transfer strength ratio (see the upper region of Fig. 4). Figure 4 shows that the energy dissipation curves of constant dissipation strength ratio have unique minimums. This means that minimum energy dissipation ice geometries are obtainable. The minimums are stable due to positive curvatures near the zero slope.

Using the velocity and temperature field solutions given in Appendix A yields a specific relationship between dissipation strength ratio and the flow and thermal control parameters as

$$\frac{\beta_W}{\beta_I} = \kappa\theta_T^2\left[1 + Br - \frac{Br^2}{12}\right]. \quad (18)$$

The Brinkman number is not zero in the Couette iceform design model. For application of the Couette model to more complicated flows, the Brinkman number can be replaced by the Nusselt number based on the water space using equation (4).

3.2. *Designation function phase plane: optimization*

The curves of constant dissipation strength ratio, β_W/β_I = constant, have minimums corresponding to the optimum geometries of minimum energy dissipation, $\zeta = \zeta_M$. Optimums are designated by the slope of the energy dissipation curves. The slope with the $\zeta = 0$ and $\zeta = 1$ singularities removed is defined as the designation function

$$d(\zeta) \equiv \zeta^2(1-\zeta)^2\frac{\partial\Phi}{\partial\zeta}. \quad (19)$$

The designation function parallels the generation function of equation (8). The designation equation, normalized by the parent surface designation magnitude, $|d(0)|$, is

$$\frac{d(\zeta)}{\beta_I} = \frac{\beta_W}{\beta_I}\zeta^2 - (1-\zeta)^2. \quad (20)$$

Figure 6 shows designation function phase plane as curves of

$$\frac{d}{\beta_I}\left(\zeta, \frac{\beta_W}{\beta_I}\right)$$

with β_W/β_I = constant. When the designation function is zero, the ice geometry minimizes flow and thermal energy dissipation. Figure 6 shows optimum ice geometries that are thermodynamically stable minimums of energy dissipation due to the positive slope of the curves. These optimum geometries can be expressed in terms of the dissipation strength ratio by finding the root of $d(\zeta_M) = 0$. For the Couette iceform, the root within $0 < \zeta_M < 1$ is

$$\zeta_M = \frac{1}{1+\sqrt{\left(\frac{\beta_W}{\beta_I}\right)}} \quad (21)$$

where the dissipation strength ratio, β_W/β_I, is given by equation (18).

In summary, the selection theory of the Couette iceform model establishes explicit expressions for $\zeta_M(Br, \theta_T)$ and $\Phi(\zeta, Br, \theta_T)$. The performance and optimum geometry are controlled by the flow and thermal parameters. Equations (18) and (21) will be used in the evolution theory.

4. EVOLUTIONARY DESIGN MODEL

The variation and selection formulations yield independent but parallel variables and equations, such as the generation and designation functions, the heat transfer strengths and energy dissipation strengths. The differences can be examined and controlled.

4.1. *The difference between steady state and minimum energy dissipation*

The iceform geometry reaches a steady state value corresponding to the heat transfer strength ratio parameter, θ_W/θ_I, as stated in equation (10). The mini-

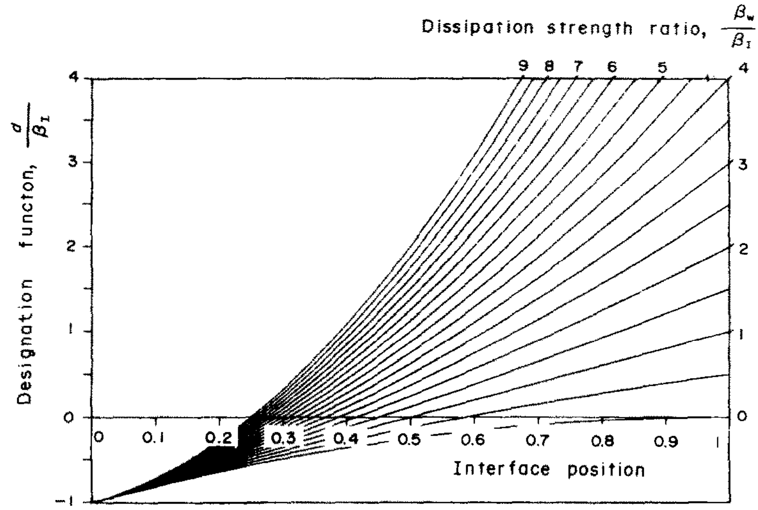

FIG. 6. Designation function phase plane.

mum energy dissipation design of the geometry is expressed in terms of the energy dissipation strength ratio parameter, β_W/β_I, as stated in equation (21). Since θ_W/θ_I is not necessarily equal to $\sqrt{(\beta_W/\beta_I)}$ the obtained steady state geometry is not necessarily equal to the optimum geometry.

Stationary states do not necessarily correspond to minimum entropy production (de Groot and Mazur [21]), and in this nonequilibrium phenomenon, the objective functional, Φ, can be stationary in time for two reasons. Using a chain rule and assuming fast relaxation ($\partial\Phi/\partial t = 0$) and fixed constraints, the time rate of change of performance is

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{\partial\Phi}{\partial\zeta}\frac{\partial\zeta}{\partial t} = \frac{d(\zeta)g(\zeta)}{\zeta^3(1-\zeta)^3}. \tag{22}$$

A stationary objective functional can be provided by $d = 0$ or $g = 0$. However, because the designation and generation functions are two independent functions, a stationary Φ in time does not necessarily correspond to optimum geometry. There are two degrees of freedom in the evolution theory.

In the experiment there are two degrees of freedom to specify, the thermal and flow control parameters. These are related to the two degrees of freedom that persist throughout this development, see Table 1.

The difference between the steady state and optimum geometries can be measured using the designation function as a 'yardstick'. The difference between steady state and optimum state is a measurable length and from this, the evolution length is defined as

$$L(\zeta_S, \zeta_M) \equiv \frac{d(\zeta_S) - d(\zeta_M)}{|d(0) - d(\zeta_M)|} \tag{23}$$

where $d(0)$ is the parent shape designation. Since $d(\zeta_M) = 0$ and $|d(0)| = \beta_I$, the evolution length can be explicitly written as

$$L\left(\zeta_S, \frac{\beta_W}{\beta_I}\right) = \frac{d(\zeta_S)}{\beta_I} = \frac{\beta_W}{\beta_I}\zeta_S^2 - (1-\zeta_S)^2 \tag{24}$$

or in terms of the strength ratios as

$$L\left(\frac{\theta_W}{\theta_I}, \frac{\beta_W}{\beta_I}\right) = \left[\frac{\beta_W}{\beta_I} - \left(\frac{\theta_W}{\theta_I}\right)^2\right]\left[1 + \frac{\theta_W}{\theta_I}\right]^{-2}. \tag{25}$$

The obtained steady state geometry is the optimum geometry if $L = 0$. This desirable condition can be specified and the corresponding flow and thermal parameters can be solved. This leads to an inverse problem.

Table 1.

1st degree of freedom	2nd degree of freedom
Form	Function
Design rate P	Performance Φ
Steady state ζ_S	Optimum ζ_M
θ_W/θ_I	β_W/β_I
θ_T	Br or Nu_I

4.2. *Inverse problem: experimental parameter specification*

The iceformation design process is controlled by two degrees of freedom, Br and θ_T. The goal of steady state design optimization, $L = 0$, is approached by specification of the experimental parameters. This is known as an inverse problem in which an interior solution is given ($L = 0$ and ζ_S) and the corresponding flow and thermal boundary conditions are sought. For instance, Burggraf [28] solved inverse conduction problems using series expansions. Additionally, Zabaras *et al.* [29] solved an inverse one-dimensional heat transfer/phase change problem (known as a Stephan problem). The inverse problem presented here contains the unknowns of thermal and flow

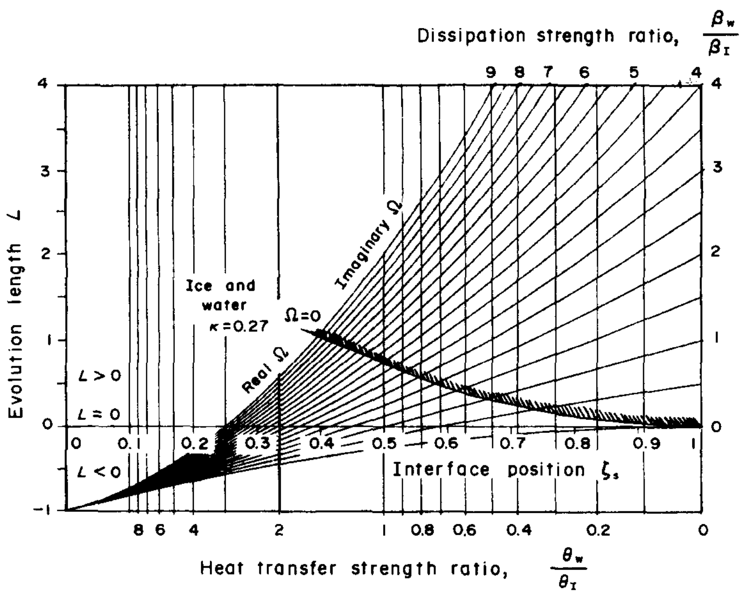

FIG. 7. Evolution length vs constraints.

boundary conditions on the upper plate given two other specifications.

The inverse problem is solved by stating the two experimental degrees of freedom, θ_T and Br, in terms of the two design and performance degrees of freedom, θ_W/θ_I and β_W/β_I. After much algebra (using equations (11) and (18))

$$Br = \frac{2\Omega\sqrt{\frac{3}{4}}}{1-\Omega\sqrt{\frac{3}{4}}} \tag{26}$$

and

$$\theta_T = \frac{1}{\kappa}\frac{\theta_W}{\theta_I}\left[1-\Omega\sqrt{\frac{3}{4}}\right] \tag{27}$$

where

$$\Omega = \sqrt{\left(1-\kappa\left(\frac{\theta_I}{\theta_W}\right)^2\frac{\beta_W}{\beta_I}\right)}. \tag{28}$$

A constraint in the design technique arises due to the necessity of Ω being a real number. This requires that

$$1-\kappa\left(\frac{\theta_I}{\theta_W}\right)^2\frac{\beta_W}{\beta_I} \geqslant 0 \tag{29}$$

or the largest value of the evolution length be bounded

$$\left(\frac{1}{\kappa}-1\right)\left(\frac{\theta_W}{\theta_I}\right)^2\zeta_S^2 \geqslant L. \tag{30}$$

This is not a problem for fluid/solid media where $0 < \kappa \leqslant 1$. In these cases, the optimum geometry at steady state, $L = 0$, is obtainable by selection of experimental constraints from a nomograph construction [30] of equations (10), (21), (26) and (30), as shown in Fig. 7. Figure 7 shows an example construction of the evolution length phase plane in terms of the inverse equations given above.

The conductivity ratio is an important quantity for the optimization goal. Many sources were reviewed to calculate κ for ice/water phase change. The value used here, $\kappa = 0.27$, was selected from a set of ten different values; therefore, the theoretical optimum of minimum energy dissipation *is obtainable* in the Couette iceform design model. Figure 7 shows a conductivity ratio constraint derived from equation (29). This shows that the iceform evolution is not constrained from reaching the optimal steady state goal. An example of this fact is shown in the flat plate iceformation paper following this theoretical model.

5. SUMMARY AND DISCUSSION

The iceformation evolution is scientifically controlled while the formation of ice is a natural process. The ice formation process generates a geometry in real time and reaches a steady state. In this real process, the changes from ζ_0 to ζ_S occur by the forces of nature. Conversely, the evolution process, which is a stepwise phenomenon, is the result of human-controlled change of experimental parameters between generations of the geometry. The motivation for experimental condition changes in a harnessed natural design process [1] is provided by the evolution length value stated in equations (23), (24) or (25).

5.1. *Two selection degrees of freedom and closure*

The selection degrees of freedom in the design evolution are found by balancing selection variables with the number of equations. The number of relevant quantities in the evolution theory is ten, ζ_S, ζ_M, θ_W/θ_I, β_W/β_I, θ_T, Br, L, Ω, T_F and U_F. The evolutionary design theory for the Couette iceform model can be summarized by an outline of eight multivariable (some nonlinear) equations

$$\theta_T \equiv \frac{T_F - T_0}{T_0 - T_R}$$

$$Br \equiv \frac{\mu U_F^2}{k_W(T_F - T_0)}$$

$$\theta_T = \frac{1}{\kappa}\frac{\theta_W}{\theta_I}\left[1 - \Omega\sqrt{\frac{3}{4}}\right]$$

$$Br = \frac{2\Omega\sqrt{\frac{3}{4}}}{1 - \Omega\sqrt{\frac{3}{4}}}$$

$$\Omega = \sqrt{\left(1 - \kappa\left(\frac{\theta_I}{\theta_W}\right)^2 \frac{\beta_W}{\beta_I}\right)}$$

$$L = \left[\frac{\beta_W}{\beta_I} - \left(\frac{\theta_W}{\theta_I}\right)^2\right]\left[1 + \frac{\theta_W}{\theta_I}\right]^{-2}$$

$$\zeta_M = \frac{1}{1 + \sqrt{\left(\frac{\beta_W}{\beta_I}\right)}}$$

$$\zeta_S = \frac{1}{1 + \frac{\theta_W}{\theta_I}}.$$

Two degrees of freedom arise because there are ten quantities linked mathematically by eight equations. The system of equations should be determinant when two selections are made and the eight equations are used. However, the nonlinear equations may prevent closure.

In any case of iceformation, two degrees of freedom must be selected to control the evolution and generation processes. This is called a preprocess determination of the experimental parameters. Two quantities can be selected from the Couette iceform theoretical variables θ_T, θ_W/θ_I, β_W/β_I, Br or L, provided that Ω is a real number. Selection substitutes appear as pairs; however, both cannot be selected. There are three substitutes

$$\zeta_S \otimes \frac{\theta_W}{\theta_I}$$

$$\zeta_M \otimes \frac{\beta_W}{\beta_I}$$

$$\Omega \otimes Br.$$

There are three restrictions to the selection of a pair. These restrictions can be shown by constructing a game (control theory) and identifying selected pairs which lead to underdeterminance. These pairs are

$$\theta_T \otimes \frac{\beta_W}{\beta_I}$$

$$\theta_T \otimes L$$

$$Br \otimes L.$$

The goal of the preprocess selection is to obtain θ_I and Br and to provide a basis for the determination of T_F and U_F to run the experiment.

5.2. *Practical considerations*

Depending on the design problem, different preprocess selections may be considered. For example, in a typical problem the optimum geometry at a certain Reynolds number is desired. This would lead to the selection of a U_F value and $L = 0$. This case would lead to $Br_M = 5.69$ and a corresponding thermal control value T_F. If T_F or U_F is selected, the process can be reversed to determine the other parameters. Some additional experimental freedom is provided if the parent surface temperature, T_R, can be adjusted and controlled. This has a direct influence on the range of the water temperature, T_F, for particular geometries, ζ_S, and fluid velocities, U_F.

The Couette iceform regime is a model for local ice formation of complicated shapes, and use of the Couette iceform model approximates the desired steady state geometry for local points. In the experiment, the parent shape geometry and cooling load are derived from local models. The theory allows the input or calculation of a local Nusselt number based on the fluid space using equation (4). Therefore, the Couette iceform model provides a basis for designing two- and three-dimensional parent surfaces using the one-dimensional Couette iceformation equations. Once the apparatus is constructed, the Couette iceform model of the interface point is used to find and specify global experimental conditions, the flow and thermal parameters, that will produce desired geometric and performance results.

6. CONCLUSIONS

The evolution theory was developed for a fundamental Couette iceform model. The theory formulates a design tool based on a one-dimensional iceformation model. The iceform geometry is related to specified flow and thermal parameters. Optimum geometries of minimum energy dissipation are obtained for certain combinations of experimental parameters.

The use of iceformation as a flow and thermal design tool requires the selection of thermal and flow boundary conditions as constraints, and these boundary constraints are linked to performance and geometry. A change in the two degrees of freedom of the constraints results in an evolution process. The goal of the evolutionary design process is to select the flow and thermal parameters such that the steady state geometry is an optimum geometry.

To use the Couette iceform model for more complicated parent surfaces and flow fields, the heat transfer and dissipation strengths must be functions of the ice/water interface position and the interface tangential coordinates. Since a complex flow requires a complex geometry, the strength ratio functions produce geometry morphology. This paper is followed by an example of the Couette iceform model on two-dimensional iceformation over a cold flat plate in a Blasius type boundary layer flow.

Further research on the iceformation design method should include experimental survey of different flow regimes. The iceformation method can be broadened by studying interface melting in a high speed wind tunnel. The geometry *and* performance monitored over time will indicate the history of the iceformation process. Computational studies that simulate the iceformation process can be compared to traditional optimization techniques to identify similarities and differences.

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APPENDIX A. SUMMARY OF COUETTE FLOW AND TEMPERATURE FIELD SOLUTIONS

The following assumptions are used in the Couette iceform model:

(1) quasi-steady design process
(2) zero pressure gradients in the near interface model

(3) small density change between phases and negligible buoyancy effects
(4) uniformity of material properties in each phase
(5) ice grows on the lower plate (see Fig. 2)
(6) the interface geometry, I, remains flat
(7) all field variables near the interface point are a function of the parent surface normal coordinate, y.

The ice and water field solutions are summarized as follows:

Ice space $\tau_I(\psi) = \psi$ where

$$\tau_I \equiv \frac{T_I - T_R}{T_0 - T_R}, \quad \psi \equiv \frac{y}{I}. \tag{31}$$

Water space $v(\lambda) = \lambda$ where

$$v \equiv \frac{u}{U_F}, \quad \lambda \equiv \frac{y-I}{\delta - I} \tag{32}$$

and

$$\tau_W(\lambda) = \lambda + \frac{Br}{2}[\lambda - \lambda^2] \tag{33}$$

where the nondimensional fluid temperature and the Brinkman number are defined by

$$\tau_W \equiv \frac{T_W - T_0}{T_F - T_0}, \quad Br \equiv \frac{\mu U_F^2}{k_W(T_F - T_0)} = Pr\, Ec. \tag{34}$$

APPENDIX B. ADJUSTABLE PERFORMANCE FUNCTIONAL DERIVATION

An arbitrary measure of performance is adopted as a basis of comparison between geometries. The performance of a geometry is governed by the fluid and thermal field orientation. The performance indicator must be consistent with the conservation laws of energy and momentum, and therefore, is formulated to correspond to the energy equation

$$k\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 = 0 \tag{35}$$

under the constraint of the equation of motion (quasi-steady formation)

$$\frac{d^2u}{dy^2} = 0. \tag{36}$$

The volumetric entropy production is an indicator of irreversibilities and performance of a design. However, the entropy production contains inverse temperature terms and is complicated to integrate. Many varied techniques are used to rectify this difficulty. One technique is to assume that the inverse temperature is equivalent to a reservoir (constant) temperature. Similarly, Bejan [22] assumed that the inverse temperature spatial variation is small in terms of absolute temperature, and a reference (constant) temperature is used. Another technique is to define the inverse temperature as 'coldness' (Ahmadi [31] and Coleman and Noll [32]). The coldness is a natural variable in entropy space and produces thermodynamic description in terms of Massieu functions (Callen [33]). In the present study, the entropy production functional is multiplied by a positive definite weighting function, T^n, which preserves the second law inequality.

Since there are two fields, T and u, and two separate terms of the energy equation (35), the adopted performance functional contains two flexible coefficients, n and m. This is sufficient to handle possible mixed terms which may arise in the formulation. The total performance functional is formulated as the integral of entropy production (irreversibility) multiplied by temperature raised to the power n

$$\Phi_T \equiv \int_0^\delta \sigma(y, T, T_y, u, u_y, m) T^n \, dy \tag{37}$$

where the entropy production rate for the Couette iceform contains both thermal and viscous dissipation as

$$\sigma = J_q(m)\frac{d}{dy}\left(\frac{1}{T}\right) + \frac{1}{T}\mu\left(\frac{du}{dy}\right)^2 \tag{38}$$

and the adjustable (m) phenomenological relation for heat flux is

$$J_q(m) = -k\frac{dT}{dy} - mu\mu\frac{du}{dy}. \tag{39}$$

However, the total performance based on entropy production includes input boundary effects due to the moving plate model. The *internal* performance indicator due to thermal and fluid orientation is the total performance minus the moving plate input (a base value)

$$\Phi = \Phi_T - U_F T_F^{n-1}\mu\left(\frac{du}{dy}\right)_{y=\delta} = \int_0^\delta \sigma T^n \, dy - U_F T_F^{n-1}\mu\left(\frac{du}{dy}\right)_{y=\delta}. \tag{40}$$

Furthermore, the boundary input causes effects within the volume

$$U_F T_F^{n-1}\mu\left(\frac{du}{dy}\right)_{y=\delta} = uT^{n-1}\mu\frac{du}{dy}\bigg|_0^\delta = \int_0^\delta \frac{d}{dy}\left[uT^{n-1}\mu\frac{du}{dy}\right] dy. \tag{41}$$

By algebraic and differential manipulation and use of the equation of motion constraint, the performance indicator due to *internal* orientation of the thermo-fluid is

$$\Phi = \int_0^\delta f(y, T, T_y, u, u_y, m, n)\, dy \tag{42}$$

where the flexible performance functional is

$$f = kT^{n-2}\left(\frac{dT}{dy}\right)^2 + uT^{n-2}\mu\frac{du}{dy}\frac{dT}{dy}(m-n+1). \tag{43}$$

The coefficients, n and m, are determined by equating the Euler–Lagrange equation for f to the governing energy equation. The Euler–Lagrange equation for the performance functional, f, is obtained using the variation of temperature, δT, and the equation of motion constraint

$$\frac{d}{dy}\left(\frac{\partial f}{\partial T_y}\right) - \frac{\partial f}{\partial T} = 0; \quad \frac{d^2u}{dy^2} = 0. \tag{44}$$

After substituting f and manipulating, the Euler–Lagrange equation becomes

$$k\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 = \left(\frac{2-n}{2}\right)\frac{k}{T}\left(\frac{dT}{dy}\right)^2 + \left(\frac{n-m+1}{2}\right)\mu\left(\frac{du}{dy}\right)^2. \tag{45}$$

The two residual terms on the right become zero by adjustment of n and m. The special values of $n = 2$ and $m = 3$ produce a performance indicator consistent with conservation of energy (equation (35)). The temperature variation, δT, corresponds to the variations in geometry, $\delta\zeta$, and therefore, the performance indicator is a function of geometry, $\Phi(\zeta)$

$$\Phi(\zeta) = \int_0^\delta \left[k\left(\frac{dT}{dy}\right)^2 + 2u\mu\frac{du}{dy}\frac{dT}{dy}\right] dy. \tag{46}$$

This performance functional, multiplied by $\frac{1}{2}$, is the introductory calculus of variations functional of energy dissipation for one-dimensional heat conduction when $u = 0$. Since the performance indicator measures the relative 'goodness' of the geometric variations, it is permissible to multiply the functional by a constant, $\frac{1}{2}$. Therefore, the performance indicator (47), is referred to as energy dissipation

$$\Phi(\zeta) = \int_0^{\delta} \left[\frac{1}{2} k \left(\frac{\mathrm{d}T}{\mathrm{d}y} \right)^2 + u\mu \frac{\mathrm{d}u}{\mathrm{d}y} \frac{\mathrm{d}T}{\mathrm{d}y} \right] \mathrm{d}y. \tag{47}$$

THEORIE DE L'EVOLUTION POUR LE CONTROLE OPTIMAL D'UN MODELE COUETTE DE FORMATION DE GLACE

Résumé—La méthode 'formation de glace' est une nouvelle technique pour concevoir la forme, à faible perte de charge, des fixations et des corps. La sélection optimale des paramètres de contrôle dynamiques et thermiques est basée sur la minimisation de la dissipation d'énergie quand l'interface glace/eau atteint un état stationnaire. Une bas-théorique est établie pour sélectionner les paramètres de contrôle. Un critère optimal est obtenu en utilisant le modèle Couette de formation de glace, pour un point sur l'interface glace/eau et une théorie d'évolution. Le mécanisme d'évolution consiste en l'évaluation de la variation dynamique et de la performance thermodynamique. On fournit une base théorique pour conduire les expériences de formation de glace et la simulation numérique des mécanismes analogues de design.

EVOLUTIONSTHEORIE FÜR DIE OPTIMALE BEEINFLUSSUNG EINES GESTALTUNGSMODELLS AUS EIS

Zusammenfassung—Die "Eisbildungs"-Methode is eine neue Technik für die Gestaltung von Gegenständen mit geringem Strömungswiderstand. Die optimale Auswahl der Parameter für die Beeinflussung der Strömung und des Wärmeübergangs beruht auf einer Minimierung der Dissipationsenergie, wenn die Eis/Wassergrenzfläche stationären Zustand erreicht hat. Für die Wahl der Parameter wird eine theoretische Grundlage geschaffen. Unter Verwendung eines Couette Eisform-Modells für einen Punkt an der Eis/Wassergrenzfläche und einer Evolutionstheorie ergibt sich ein optimales Kriterium. Der Evolutionsprozeß besteht aus einer dynamischen Variation und der Auswertung des thermodynamischen Verhaltens. Abschließend wird eine theoretische Grundlage für die Ausführung von Eisbildungsexperimenten und für die numerische Simulation analoger Gestaltungsprozesse gegeben.

ИСПОЛЬЗОВАНИЕ ЭВОЛЮЦИОННОЙ ТЕОРИИ ДЛЯ ОПТИМАЛЬНОГО РЕГУЛИРОВАНИЯ МОДЕЛИ КУЭТА ПРИ ОБРАЗОВАНИИ ЛЬДА

Аннотация—Метод "льдообразования" предоставляет собой новый способ используемый при разработъе формы креплений и тел в случаях течений с низкими потерями. В основе оптимального выбора параметров течения и тепловой регуляции лежит минимизация рассеяния энергии при достижении границей раздела лед/вода стационарного состояния. Установлен теоретический принцип выбора определяющих параметров. Оптимальный критерий получен с использованием модели Куэтта для точки на границе раздела лед – вода. Исследуемый эволюционный процесс позволяет оценить динамическое изменение и термодинамические характеристики. Даются теоретические основы проведения экспериментов по льдообразованию и численного моделирования аналогичных процессов.